ESERCIZI SULLE SERIE NUMERICHE

Stabilire quali delle seguenti serie convergono:

(a) $\sum_{n=1}^{+\infty} \log\left(1+\frac{1}{n}\right)\left(\sqrt{n^2+2}-n\right)$ (b) $\sum_{n=1}^{+\infty} \left(n-\sqrt{n-1}\right)\left(1-e^{-\frac{1}{n^2}}\right)$

(c) $\sum_{n=1}^{+\infty} n^4 e^{-\sqrt{n}}$ (d) $\sum_{n=1}^{+\infty} \frac{3(\sin(\sqrt{n})^2-1}{3n^2+(-1)^n}$

(e) $\sum_{n=1}^{+\infty} \frac{n!}{(2n)!}$ (f) $\sum_{n=1}^{+\infty} (-1)^n \log\left(1-\frac{1}{n}\right)$

(g) $\sum_{n=1}^{+\infty} \left(e^{\frac{1}{\sqrt{n}}}+1-2\cos\left(\frac{1}{\sqrt{n}}\right)\right)$ (h) $\sum_{n=1}^{+\infty} \frac{\sin\left(\frac{1}{n}\right)\left(\sqrt{n^2+1}-n\right)}{e^{\frac{1}{n}}+1}$

Dire per quali valori del parametro α le seguenti serie convergono:

(i) $\sum_{n=1}^{+\infty} \frac{e^{\tan\left(\frac{1}{n}\right)}-1}{n^2+n^\alpha}, \quad \alpha \in \mathbb{R}$

(j) $\sum_{n=1}^{+\infty} \frac{\sin\left(\frac{1}{\sqrt{n}}\right)}{\sqrt{n}+n^\alpha}, \quad \alpha \in \mathbb{R}$

(k) $\sum_{n=1}^{+\infty} \frac{\sqrt{n^2+n^\alpha}-n}{\log n}, \quad \alpha \in \mathbb{R}$

(l) $\sum_{n=1}^{+\infty} \frac{\arctan(n)}{(n^\alpha+n)\left(e^{\frac{1}{n}}-1\right)}, \quad \alpha \in \mathbb{R}$

(m) $\sum_{n=1}^{+\infty} \left(e^{\frac{1}{n}}-1-\frac{1}{n}\right)\sin\left(\frac{1}{n^\alpha}\right)(n+n^\alpha), \quad \alpha \in \mathbb{R}$

(n) $\sum_{n=1}^{+\infty} \frac{\alpha^n+n^\alpha}{n^3+n\log n}, \quad \alpha \in \mathbb{R}^+$

(o) $\sum_{n=1}^{+\infty} \frac{e^{\frac{1}{n^2}}}{\left(1+\frac{1}{n}\right)^n\left(1-\cos\left(\frac{1}{n^\alpha}\right)\right)n^2}, \quad \alpha \in \mathbb{R}^+.$

Soluzioni (SI = converge, NO = non converge): (a) SI, (b) NO, (c) SI, (d) SI, (e) SI, (f) SI, (g) NO, (h) SI (i) $\forall\alpha \in \mathbb{R}$, (j)$\alpha > 1/2$, (k) $\alpha < 0$, (l) $\alpha > 2$, (m) $\alpha > 0$, (n) $0 \leq \alpha \leq 1$, (o) $\alpha < 1/2$.